SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH 2021

Part – III

Time : 2 Hours

MATHEMATICS

Cool-off time : 20 Minutes

Maximum : 60 Scores

Part - A

Questions from 1 to 10 carry 3 scores each. **(10 × 3 = 30)**

1. Find the values of x for which

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

2. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   (i) Find adj A **(2)**

   (ii) Find A.adj A. **(1)**

3. Find the value of k so that the function

$$f(x) = \begin{cases} kx + 1 & , \text{if } x \le 5 \\ 3x - 5 & , \text{if } x > 5 \end{cases}$$

is continuous at $x = 5$. **(3)**

4. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$. **(3)**

5. Find the rate of change of the area of a circle with respect to its radius r when r = 5 cm. **(3)**

6. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$. **(3)**

7. Find the equation of a plane passing through the point (1, 4, 6) and the normal to the plane is $\hat{i} - 2\hat{j} + \hat{k}$. **(3)**

8. (i) Which of the following can be the domain of the function $\cos^{-1}x$?

   (a) $(0, \pi)$ (b) $[0, \pi]$

   (c) $(-\pi, \pi)$ (d) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ **(1)**

   (ii) Find the value of $\cos^{-1}(-1/2) + 2\sin^{-1}(1/2)$. **(2)**

9. Find the area of a triangle with vertices (–2, –3), (3, 2) and (–1, –8). **(3)**

10. Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x$. **(3)**

(12 × 4 = 48)

Questions from 11 to 22 carry 4 scores each.

11. Consider the matrices $A = \begin{bmatrix} 3 & 4 \\ -5 & -1 \end{bmatrix}$ and $3A + B = \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix}$

   (i) Find the matrix B. **(2)**

   (ii) Find AB. **(2)**

12. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ and $B = [1, 3, -6]$

   (i) What is the order of AB ? **(1)**

   (ii) Verify $(AB)' = B'A'$. **(3)**

13. (i) If $xy < 1$, $\tan^{-1} x + \tan^{-1} y =$ _______.

(a) $\tan^{-1}\frac{x-y}{1+xy}$ (b) $\tan^{-1}\frac{1-xy}{x+y}$

(c) $\tan^{-1}\frac{x+y}{1-xy}$ (d) $\tan^{-1}\frac{x+y}{1+xy}$ **(1)**

(ii) Prove that $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$. **(3)**

14. Find $\frac{dy}{dx}$

(i) $x^2 + xy + y^2 = 100$. **(2)**

(ii) $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $-1 \le x \le 1$. **(2)**

15. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is

(i) increasing

(ii) decreasing **(4)**

16. (i) Find the order and degree of the differential equation $\left(\frac{ds}{dt}\right)^4 + \frac{3\,d^2s}{dt^2} = 0$. **(1)**

(ii) Find the general solution of the differential equation $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$. **(3)**

17. Find a unit vector both perpendicular to the vectors if

$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$.

18. Find the shortest distance between the skew lines

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\ (2\hat{i} + \hat{j} + 2\hat{k})$

19. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$. Find (2)

(i) $P(A \cap B)$ (1)

(ii) $P(A|B)$ (1)

(iii) $P(A \cup B)$

20. (i) Let R be a relation on a set $A = \{1, 2, 3\}$, defined by $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Then the ordered pair to be added to R to make it a smallest equivalence relation is _______.

(a) (2, 1) (b) (3, 1)

(c) (1, 2) (d) (1, 3) (1)

(ii) Determine whether the relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(x, y) : y$ is divisible by $x\}$ is reflexive, symmetric and transitive. (3)

21. Find $\frac{dy}{dx}$

(i) x^x (2)

(ii) $x = 2at^2$; $y = at^4$ (2)

22. Integrate :

$$\int \frac{x}{(x+1)(x+2)} dx.$$

23. (i) Construct a 3×2 matrix $A = [a_{ij}]$ whose elements are given by (2)

$$a_{ij} = 3\hat{i} - \hat{j}$$

(ii) Express $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. (4)

24. Solve the following system of equations by matrix method

$3x - 2y + 3z = 8$

$2x + y - z = 1$ **(6)**

$4x - 3y + 2z = 4$

25. (i) Let $f : \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g : \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof. **(3)**

(ii) Consider $f : R \rightarrow R$ given by $f(x) = 2x + 1$. Show that f is invertible. Find the inverse of f. **(3)**

26. (i) Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$. **(2)**

(ii) Find the equation of tangent to the above curve. **(2)**

(iii) What is the maximum value of the function $\sin x + \cos x$? **(2)**

27. Integrate :

(i) $\int \sin x \sin (\cos x)\, dx.$

(ii) $\int_0^1 \frac{\tan^{-1}x}{1 + x^2}\, dx.$

28. Solve the following problem graphically

Maximise : $z = 3x + 2y$

Subject to : $x + 2y \leq 10$

$3x + y \leq 15,$ **(6)**

$x, y \geq 0$

29. (i) Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$ and $x = 4$ and the x-axis. **(3)**

(ii) Find the area of the region bounded by two parabolas $y = x^2$ and $y^2 = x$. **(3)**